

**Total Marks – 120****Attempt Questions 1-10****All questions are of equal value**

Answer each question in a SEPARATE writing booklet.

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<b>Question 1</b>	<i>(12 Marks)</i>	<b>Marks</b>
(a)	Find the value of $e^3$ correct to three significant figures.	2
(b)	Find integers $a$ and $b$ such that $(3 - \sqrt{5})^2 = a + b\sqrt{5}$ .	2
(c)	Find the exact value of $\sin \frac{\pi}{4} - \cos \frac{3\pi}{4}$ .	2
(d)	Find the values of $x$ for which $ x + 2  < 3$ .	2
(e)	Find a primitive of $1 + \frac{1}{x}$ .	2
(f)	Simplify $\frac{x}{x^2 - 9} - \frac{1}{x - 3}$ .	2

**Question 2** *(12 Marks)* Use a SEPARATE writing booklet. $A, B$  and  $C$  are the points  $(-2, -1)$ ,  $(0, 3)$  and  $(4, 0)$  respectively.

(a)	Sketch the $\triangle ABC$ on the number plane.	1
(b)	Find the gradient of the line $BC$ .	1
(c)	Show that the equation of the line $BC$ is $3x + 4y - 12 = 0$ .	1
(d)	Find the coordinates of $D$ , the midpoint of $AC$ .	1
(e)	$D$ is the midpoint of the interval $BE$ . Find the coordinates of $E$ .	2
(f)	What type of quadrilateral is $ABCE$ ? Give a reason for your answer.	2
(g)	Find the perpendicular distance from the point $A$ to the line $BC$ .	2
(h)	Find the length of $BC$ and hence find the area of quadrilateral $ABCE$ .	2

**Question 3** (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Differentiate with respect to  $x$ : **5**

(i)  $(2x - 5)^3$

(ii)  $\log_e(x^2 - 5)$

(iii)  $\frac{x}{e^x}$

(b) (i) Find  $\int \frac{dx}{x^2}$  **3**

(ii)  $\int_0^{\pi/6} \sec^2 x dx$

(c) The equation  $(x - 3)^2 = -8(y + 2)$  represents a parabola. **4**

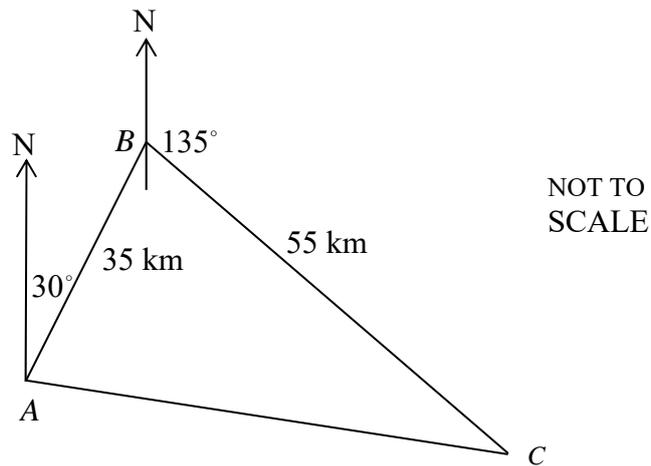
(i) Write down the coordinates of the vertex.

(ii) What is the focal length of the parabola?

(iii) Sketch the graph of the parabola, clearly indicating the focus and the directrix on your diagram.

**Question 4** (12 Marks) Use a SEPARATE writing booklet. **Marks**

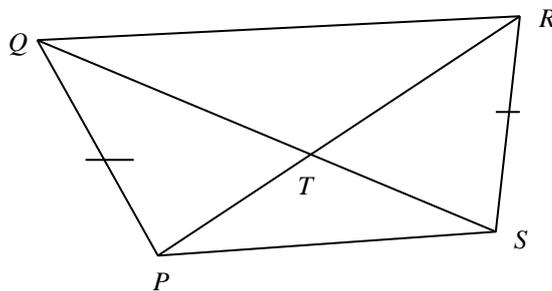
- (a) A motorist drives 35 km from Town  $A$  to Town  $B$  on a bearing of  $030^\circ$ . **4**  
 He then drives 55 km to Town  $C$  that is on a bearing of  $135^\circ$  from  
 Town  $B$ .



- (i) Find the size of  $\angle ABC$ .
- (ii) Find the distance between  $A$  and  $C$  to the nearest kilometre.
- (b) Three manufacturing plants  $A$ ,  $B$  and  $C$  supply, respectively, 20%, 30% and 50% of all shock absorbers used by a certain automobile manufacturer. Records show that the percentage of defective items produced by  $A$ ,  $B$  and  $C$  is 3%, 2% and 1% respectively. What is the probability that a randomly chosen shock absorber installed by the manufacturer will be defective? **3**
- (c) The first term of an arithmetic series is 4 and the 5<sup>th</sup> term is four times the 3<sup>rd</sup> term. Find the common difference. **3**
- (d) Sketch a graph of the function  $y = \log_e(x-1)$  showing all important features. **2**

**Question 5** (12 Marks) Use a SEPARATE writing booklet. **Marks**

- (a) The region bounded by the curve  $y = \frac{1}{\sqrt{1+x}}$  and the  $x$ -axis between  $x = 0$  and  $x = 2$  is rotated about the  $x$ -axis. Find the volume of the solid that is formed. **3**
- (b) The diagonals  $PR$  and  $QS$  of the quadrilateral  $PQRS$  are equal and intersect at  $T$ . Also  $PQ = RS$ . **7**



NOT TO  
SCALE

- (i) Show that  $\triangle PSR \equiv \triangle SPQ$ .
- (ii) Hence show that  $\triangle PST$  is isosceles.
- (iii) Show that  $\triangle QTR$  is also isosceles.
- (iv) Show that  $PS$  is parallel to  $QR$ .
- (c) (i) Write down the discriminant of  $2x^2 - 5x + k$ . **2**
- (ii) For what values of  $k$  does  $2x^2 - 5x + k = 0$  have real roots?

**Question 6** (12 Marks) Use a SEPARATE writing booklet. **Marks**

Consider the function  $f(x) = 4xe^{-2x}$ .

- (a) State the  $y$ -intercept of the function. **1**
- (b) Show that  $f'(x) = 4e^{-2x}(1 - 2x)$ . **2**
- (c) State the coordinates of the stationary point. **2**
- (d) Use the fact that  $f''(x) = 16e^{-2x}(x - 1)$  to determine the nature of the stationary point. **1**
- (e) Find the coordinates of any points of inflexion. **2**
- (f) Find  $\lim_{x \rightarrow \infty} f(x)$  and  $\lim_{x \rightarrow -\infty} f(x)$ . **2**
- (g) Hence sketch a graph of  $y = f(x)$  indicating all important features. **2**

**Question 7** (12 Marks) Use a SEPARATE writing booklet.

- (a) (i) Sketch the parabola  $y = x^2 - x - 2$ , clearly labelling all intercepts with the axes. **5**
- (iii) On the same number plane, sketch the line  $y = x - 2$ .
- (iv) Find the area bounded by the parabola and the line.
- (b) A baby whale is growing at a rate proportional to its weight  $W$  kg. **5**  
That is,  $\frac{dW}{dt} = kW$  for some constant  $k$ .
- (i) Show that  $W = W_0 e^{kt}$  is a solution to the differential equation.
- (ii) Given that the whale weighed 40 kg at birth and 60 kg when it was one month old, find the exact values of  $W_0$  and  $k$ .
- (iii) Find, to the nearest kilogram, the whale's weight when it is one year old.

*Question 7 continues on Page 6...*

**Question 7 continued****Marks**

- (c) An object is in motion along a line. The velocity, as measured at several instants of time, is given in the following table. Use the trapezoidal rule to approximate the distance travelled from  $t = 0$  to  $t = 6$  seconds. **2**

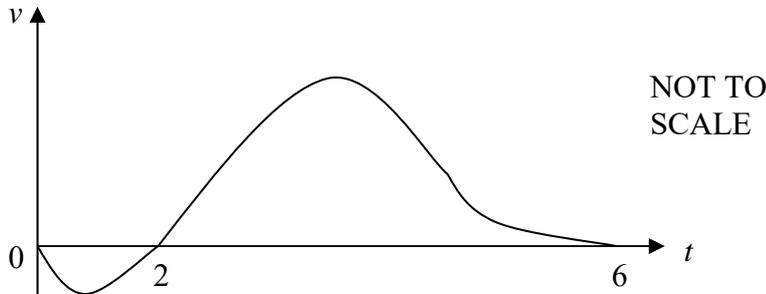
$t$ (s)	0	1	2	3	4	5	6
$v$ (m/s)	3.2	2.7	2.9	4.0	4.7	5.6	5.7

**Question 8** (12 Marks) Use a SEPARATE writing booklet.

- (a) Amy had a full drink bottle containing 500 ml of water. She drank from it so that the volume  $V$ , in millilitres, of water in the bottle changed at a rate given by  $\frac{dV}{dt} = \frac{2}{5}t - 20$  ml/s. **7**
- (i) Find a formula for  $V$ .
  - (ii) Show that it took Amy 50 seconds to drink the contents of the bottle.
  - (iii) How long, to the nearest second, did it take Amy to drink half the contents of the bottle?
- (b) Max makes a payment of  $\$M$  into a superannuation fund at the start of each year for 25 years. Interest is compounded annually at a rate of 8% per annum. **5**
- (i) Give an expression for the amount that the first instalment is worth after 25 years.
  - (ii) Find, in terms of  $M$ , the value of the total investment at the end of the 25<sup>th</sup> year.
  - (iii) At the end of 25 years, Max's superannuation amounts to \$650 000. Find the value of  $M$ .

**Question 9** (12 Marks) Use a SEPARATE writing booklet.**Marks**

- (a) A particle is observed as it moves in a straight line in the period between  $t = 0$  and  $t = 6$ . Its velocity  $v$  at time  $t$  is shown on the graph below. **5**



- (i) Copy this diagram onto your paper and clearly label with the letter *A* the times when the acceleration of the particle is zero.
- (ii) State the times at which the particle is at rest. Giving reasons for your answer, state on which of these occasions the particle is furthest from its initial position.
- (iii) Assuming that the particle starts from the origin, sketch a graph of displacement against time.
- (b) (i) On the same set of axes accurately draw the graphs of  $y = 2 \sin 3x$  and  $y = 2 - x$  for  $0 \leq x \leq \pi$ . **4**
- (ii) How many solutions are there to the equation  $2 \sin 3x = 2 - x$ ?
- (c) Solve  $2 \cos^2 x - 3 \cos x - 2 = 0$  for  $0 \leq x \leq 2\pi$ . **3**

**Question 10** (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Show that if  $y = \frac{e^x - e^{-x}}{2}$ , then  $x = \log_e \left( y + \sqrt{y^2 + 1} \right)$ . **3**

(b) For a certain kind of telegraph cable the speed  $s$  of the signal is given **4**  
by  $s = kx^2 \log_e \left( \frac{1}{x} \right)$  where  $k$  is a positive constant and  $x$  is the ratio of  
the radius of the core to the thickness of the covering. Determine the  
value of  $x$  for which the maximum speed occurs.

(c) (i) Find the sum of the arithmetic series  $1 + 2 + 3 + \dots + 30$ . **5**

(ii) Show that  $\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots + \frac{n}{n} = \frac{n+1}{2}$ .

(iii) Hence find the sum of the first 465 terms of the series:

$$\frac{1}{1} + \frac{1}{2} + \frac{2}{2} + \frac{1}{3} + \frac{2}{3} + \frac{3}{3} + \frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \frac{4}{4} + \dots$$

**END OF PAPER**

(a)  $e^3 = 20.1$  correct  $\rightarrow$  3 sig figs. ✓ answer  
 ✓ rounding

(b)  $(3-\sqrt{5})^2 = 14 - 6\sqrt{5}$   
 $\therefore a = 14$  ✓ and  $b = -6$  ✓

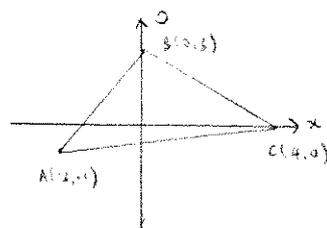
(c)  $\sin \frac{\pi}{4} - \cos \frac{3\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$  ✓✓

(d)  $|x - (-2)| < 3$   
 $\therefore -5 < x < 1$  ✓✓

(e)  $x + 10 > e^x$  ✓

(f)  $\frac{x}{(x-3)(x+3)} - \frac{1}{x-3} = \frac{x - (x+3)}{(x-3)(x+3)}$  ✓  
 $= \frac{-3}{(x-3)(x+3)}$  ✓

(a)



(b)  $m_{BC} = -\frac{5}{4}$  ✓

(c)  $y - 3 = -\frac{3}{4}x$  ✓

$4y - 12 = -3x$

$3x + 4y - 12 = 0$

(d)  $D(1, -\frac{1}{2})$  ✓

(e)  $B(0, 3)$   $D(1, -\frac{1}{2})$   $E(2, 0)$

$\frac{x+0}{2} = 1 \therefore x = 2$

$\frac{y+3}{2} = -\frac{1}{2} \therefore y = -4 \therefore E(2, -4)$

(f) parallelogram ✓

diagonals bisect one another ✓

(g)  $d = \frac{|-6-4-12|}{\sqrt{9+16}} = \frac{22}{5}$  ✓

(h)  $d_{BC} = 5$  ✓

Area of ABCE =  $5 \times \frac{22}{5} = 22 \text{ units}^2$  ✓

$$e^3 \approx 20.1 \text{ to } 3 \text{ s.f. } \checkmark \checkmark$$

$$(3 - \sqrt{5})^2 = 14 - 6\sqrt{5}$$

$$\therefore a = 14, b = -6$$

$$\sin \frac{\pi}{4} - \cos \frac{3\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} \checkmark$$

$$|x - (-2)| < 3 \quad \therefore -5 < x < 1 \checkmark \checkmark$$

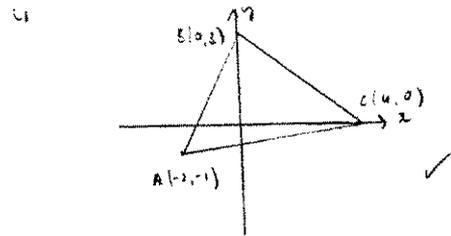
$$x + \log_e x$$

$$\frac{x}{(x-3)(x+3)} - \frac{1}{x-2}$$

$$= \frac{x - (x+3)}{(x-3)(x+3)}$$

$$= \frac{-3}{(x-3)(x+3)}$$

### Question Two



$$(b) m_{BC} = -\frac{3}{4} \checkmark$$

$$(c) y - 3 = -\frac{3}{4}(x - 0) \checkmark$$

$$4y - 12 = -3x$$

$$3x + 4y - 12 = 0$$

$$(d) \left( \frac{-2+4}{2}, \frac{-1+0}{2} \right) = \left( 1, -\frac{1}{2} \right) \checkmark$$

$$(e) B(0, 3) \quad D\left(1, -\frac{1}{2}\right) \quad E(x, 0)$$

$$\frac{x+0}{2} = 1 \quad \frac{3+0}{2} = -\frac{1}{2}$$

$$x = 2 \quad y = -4$$

$$\therefore E(2, -4) \checkmark$$

(f) Parallelogram - diagonals bisect each other

$$(g) A(-2, -1) \quad C(4, 0) \quad 3C = 3x + 4y - 12 = 0$$

$$x = \frac{1 - 4 - 12}{3} = -\frac{22}{3} \checkmark$$

$$\therefore \text{Area } ABCE = 22 \text{ units}^2 \checkmark$$

### Question Three

$$(a) (i) \frac{d}{dx} (2x-5)^3 = 6(2x-5)^2 \checkmark \checkmark$$

$$(ii) \frac{d}{dx} \log_e(x^2-5) = \frac{2x}{x^2-5} \checkmark$$

$$(b) \frac{d}{dx} \left( \frac{x}{e^x} \right) = \frac{e^x - x e^x}{e^{2x}} \checkmark \checkmark$$

$$= \frac{1-x}{e^x}$$

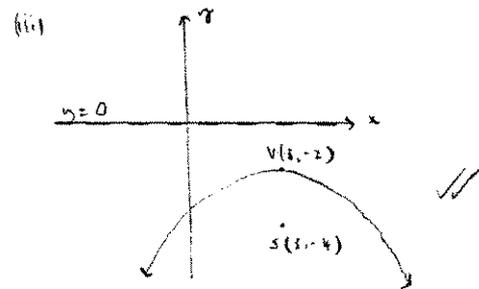
$$(b)(i) \int \frac{1}{x^2} = -\frac{1}{x} + c \checkmark$$

$$(ii) \int_0^{\frac{\pi}{3}} \sec^2 x \, dx = \left( \tan x \right)_0^{\frac{\pi}{3}} \checkmark$$

$$= \frac{1}{\sqrt{3}} \checkmark$$

$$(c) (i) V(2, 2) \checkmark$$

$$(ii) a = 2 \checkmark$$



### Question Four

$$(a) (i) \angle ABC = 75^\circ \checkmark$$

$$(ii) d_{AC}^2 = 35^2 + 55^2 - 2 \times 35 \times 55 \times \cos 75^\circ \checkmark \checkmark$$

$$d \approx 57 \text{ km} \checkmark$$

$$\text{plant A} \neq \text{def. } \frac{2}{10} \times \frac{3}{100} = \frac{3}{1000} \checkmark$$

$$\text{plant B} \neq \text{def. } \frac{3}{10} \times \frac{2}{100} = \frac{3}{1000} \checkmark$$

$$\text{plant C} \neq \text{def. } \frac{1}{2} \times \frac{1}{100} = \frac{1}{200}$$

$$(c) a = 4$$

$$a + 4d = 4(a + 2d) \checkmark$$

$$a + 4d = 4a + 8d$$

$$-3d = 3a = 12 \checkmark$$

$$\therefore \frac{3}{2500} + \frac{3}{800} + \frac{1}{200} = \frac{17}{100}$$

$$d = -3 \checkmark$$

Q4.

(a) i)  $\angle ABC = 75^\circ$  ✓

ii)  $AC^2 = 35^2 + 55^2 - 2 \times 35 \times 55 \cos 75^\circ$  ✓

$AC = \sqrt{3253.55}$  ✓

$= 57 \text{ km.}$  ✓

(b) plant A & det.  $\frac{2}{10} \times \frac{3}{100} = \frac{3}{500}$

plant B & det.  $\frac{3}{10} \times \frac{2}{100} = \frac{3}{500}$  ✓

plant C & det.  $\frac{1}{2} \times \frac{1}{100} = \frac{1}{200}$

$\therefore \frac{3}{500} + \frac{3}{500} + \frac{1}{200} = \frac{17}{1000}$  ✓

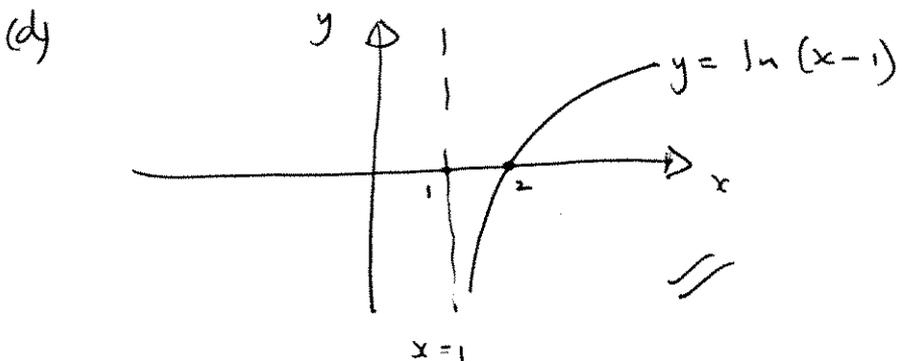
(c)  $a = 4$   $a + 4d = 4(a + 2d)$  ✓

$a + 4d = 4a + 8d$

$4d = 12 + 8d$

$4d = -12$  ✓

$d = -3$  ✓



Q5

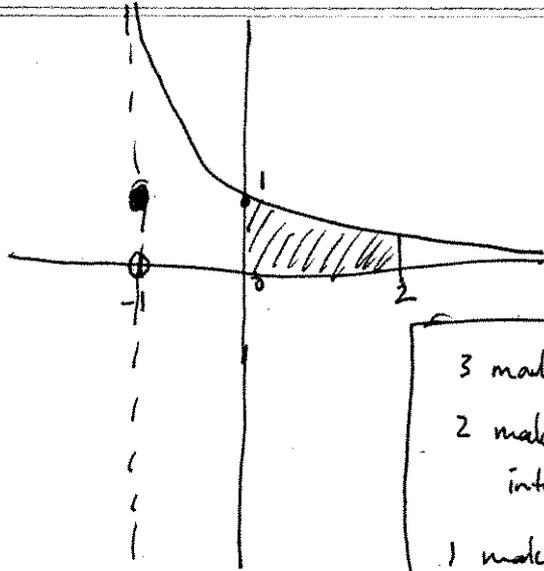
(a)  $y = \frac{1}{\sqrt{1+x}}$

$y^2 = \frac{1}{1+x}$

Vol =  $\pi \int_0^2 \frac{dx}{1+x}$

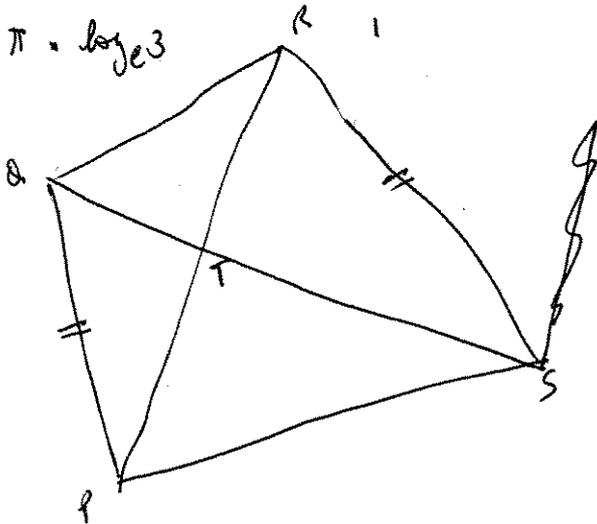
=  $\pi [\log_e(1+x)]_0^2$

=  $\pi \cdot \log_e 3$



3 marks correct ans  
2 marks for correct integration  
1 mark for correct method

(b)



PR = QS

PQ = RS

(i) In  $\triangle PQS, SRP$   
QS = PR (given)  
PQ = RS (given)  
PS is common

2 mks correct  
1 mks no reason

$\therefore \triangle PQS = \triangle SRP$  (SSS).

(ii)  $\angle RPS = \angle QSP$  (matching angles in congruent triangles equal)  
 $\therefore \triangle PST$  is isosceles. (base angles equal).

1 mks correct

(iii) Since  $PT = TS$  ( $\triangle PST$  is isosceles) (equal sides opp equal angles).  
Hence  $PR = QS$

Then  $QT = QS - TS$

$\therefore QT = PR - PT$  (from parts (i) and (ii))

$\therefore QT = RT$

$\therefore \triangle QTR$  is isosceles.

2 mks correct ans  
1 mks no reason

(iv)  $\angle QRT = \frac{1}{2}(180 - \angle QTR)$  (angle sum of isosceles  $\triangle$ )  
 $= \frac{1}{2}(180 - \angle PTS)$  (vert opp angles)  
 $= \angle TPS$  (angle sum of isosceles  $\triangle$ )

2 mks correct  
1 mks no reason

$\therefore QR \parallel PS$  (alt angles equal).

(c) (i)  $\Delta = (-5)^2 - 4(2)(k)$

(ii)  $25 - 8k \geq 0$

$\therefore \Delta = 25 - 8k$

$\therefore 8k \leq 25$

$\therefore k \leq \frac{25}{8}$

1 mks

1 marks

Q6

$$f(x) = 4x e^{-2x}$$

(a) let  $x=0$   $\therefore y=0$ . ie  $(0,0)$ . 1mk.

(b)  $f'(x) = 4e^{-2x} - 8x e^{-2x}$   
 $= 4e^{-2x}(1-2x)$ . 1mk.  
 (factorisation)  $\left\{ \begin{array}{l} u = 4x \\ u' = 4 \\ v = e^{-2x} \\ v' = -2e^{-2x} \end{array} \right.$  1mk. (product rule)

(c) If  $f'(x) = 0$  then  $4e^{-2x}(1-2x) = 0$

$\therefore 2x = 1$  1mk (x-ordinate)  
 $x = 1/2$

If  $x=1/2$  then  $y = 4(1/2)e^{-2(1/2)}$   
 $\therefore y = 2e^{-1}$ . 1mk. (y-ordinate).  
 $\therefore$  st. point at  $(1/2, 2/e)$ .

(d)  $f''(x) = 16e^{-2x}(x-1)$

$\therefore f''(1/2) = 16e^{-1}(1/2-1) < 0$   $\therefore$  max t.p. at  $(1/2, 2/e)$ .  
1mk (for sign)  $\leftarrow$

(e) let  $f''(x) = 0$   $\therefore x = 1$  1mk (x-ordinate)

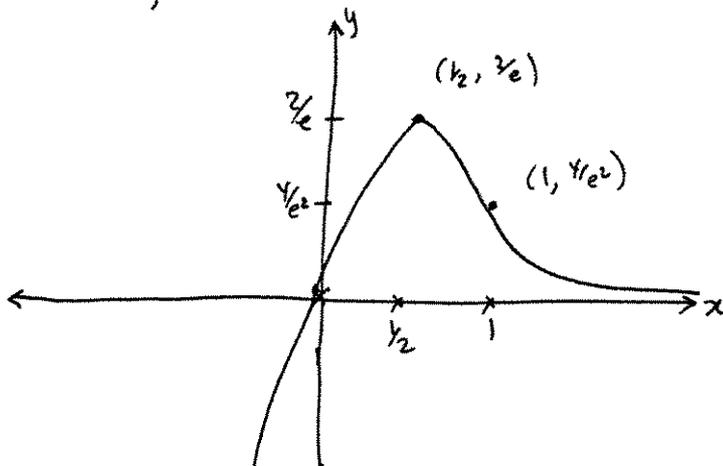
If  $x=1$ ;  $y = 4(1)e^{-2} = 4e^{-2}$   
 $\therefore$  pt of inflexion at  $(1, 4/e^2)$ . 1mk (y-ordinate).

(f) As  $x \rightarrow \infty$ ,  $e^{-2x} \rightarrow 0$   $\therefore 4x e^{-2x} \rightarrow 0^+$ . 1mk

$\therefore \lim_{x \rightarrow \infty} 4x e^{-2x} = 0$

As  $x \rightarrow -\infty$ ,  $e^{-2x} \rightarrow \infty$   $\therefore 4x e^{-2x} \rightarrow -\infty$ . 1mk

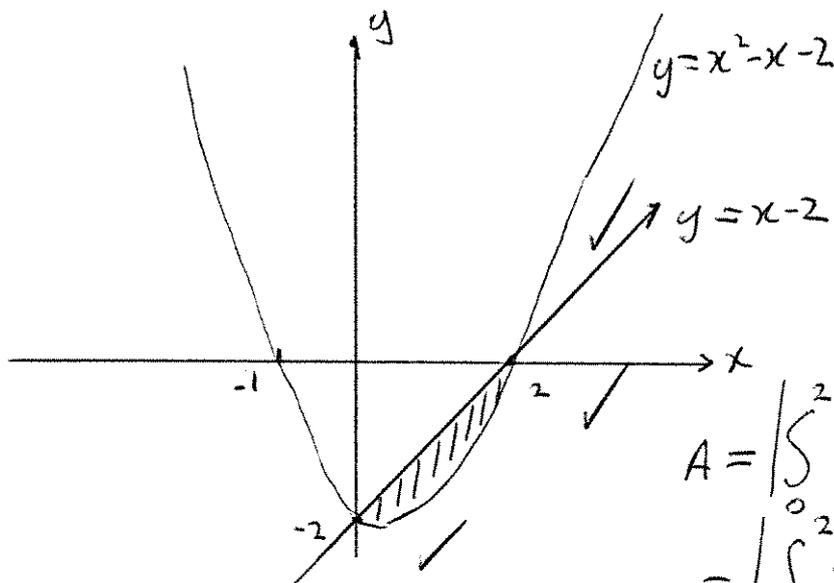
(g)



1mk max & pt of inf.  
1mk intercept & limits.

Question 7

a)



$$y = x^2 - x - 2$$

$$= (x-2)(x+1)$$

$$A = \left| \int_0^2 (x-2) - (x^2-x-2) dx \right|$$

$$= \left| \int_0^2 (x-2-x^2+x+2) dx \right|$$

$$= \left| \int_0^2 (-x^2+2x) dx \right|$$

$$= \left| \left[ -\frac{x^3}{3} + x^2 \right]_0^2 \right|$$

$$= \left| \left( -\frac{8}{3} + 4 \right) - (0) \right|$$

$$= \left| \frac{1}{3} \text{ units}^2 \right|$$

(5)

b) i)  $W = W_0 e^{kt}$

$$\frac{dW}{dt} = kW_0 e^{kt}$$

$$= kW$$

ii)  $t=0 \quad W=40$

$$40 = W_0 e^0$$

$$W_0 = 40$$

$$W = 40 e^{kt}$$

$$60 = 40 e^k$$

$$e^k = \frac{3}{2}$$

$$k = \ln \frac{3}{2}$$

iii)  $t=12$

$$W = 40 e^{\ln \frac{3}{2} \times 12}$$

$$= 40 e^{12 \ln \frac{3}{2}}$$

$$= 5189.853515$$

$$= 5190 \text{ kg (nearest kg)}$$

c)  $0 \approx \frac{1}{2} [3.2 + 5.7 + 2(2.7 + 2.9 + 7.0 + 4.7 + 5.6)]$

$$= 24.35 \text{ m}$$

(2)

## Question 8

$$a) i) \frac{dV}{dt} = \frac{2}{5}t - 20 \quad \checkmark$$

$$V = \int \left( \frac{2}{5}t - 20 \right) dt \quad \checkmark$$
$$= \frac{1}{5}t^2 - 20t + C$$

$$t=0 \quad V=500$$

$$500 = 0 - 0 + 500 \quad \checkmark$$

$$\therefore V = \frac{1}{5}t^2 - 20t + 500$$

$$ii) t = 50$$

$$V = \frac{1}{5}(50)^2 - 20(50) + 500$$
$$= 500 - 1000 + 500 \quad \checkmark$$
$$= 0$$

$$iii) V = 250$$

$$250 = \frac{1}{5}t^2 - 20t + 500$$

$$0 = \frac{1}{5}t^2 - 20t + 250 \quad \checkmark$$

$$0 = t^2 - 100t + 1250$$

$$t = \frac{+100 \pm \sqrt{(-100)^2 - 4 \times 1 \times 1250}}{2 \times 1}$$

$$= \frac{+100 \pm \sqrt{5000}}{2} \quad \checkmark$$

$$= 85.3 \dots \text{ or } 14.67 \dots \quad \checkmark$$

$$t = 15 \text{ sec (nearest sec)}$$

(7)

$$b) i) A = \left(1 + \frac{3\%}{100}\right)^{25} M$$
$$= \$M(1.08)^{25} \quad \checkmark$$

$$ii) A = \$m \left( (1.08)^{25} + (1.08)^{24} + \dots + (1.08)^1 \right) \quad \checkmark$$

$$= \$m \left( \frac{1.08(1.08^{25} - 1)}{1.08 - 1} \right) \quad \checkmark$$

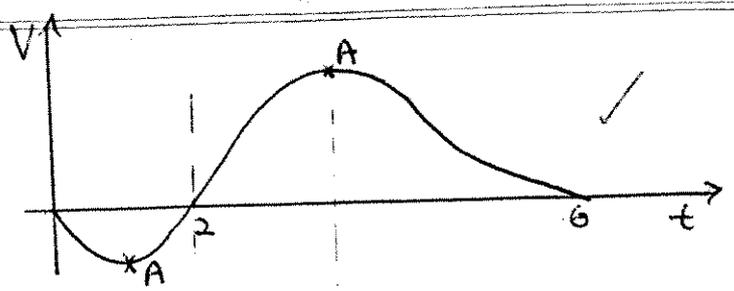
$$= \$78.95441515 \times m \quad \checkmark$$

(5)

$$iii) \$78.95441515m = \$650,000$$

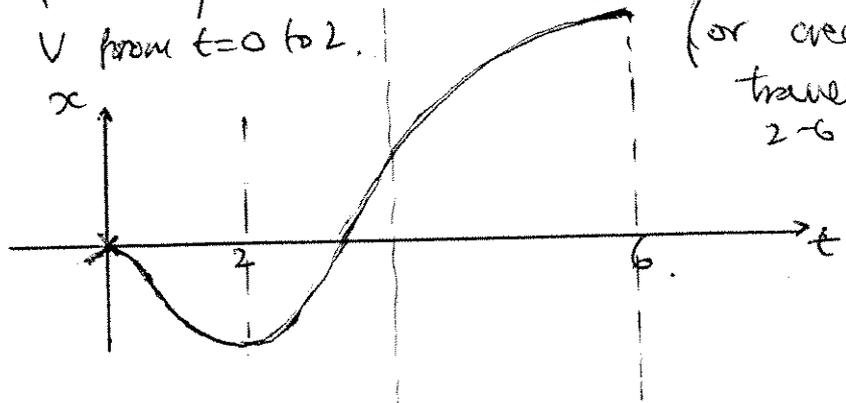
$$m = \$8232.60 \text{ (nearest cent)} \quad \checkmark$$

29(a) (i)



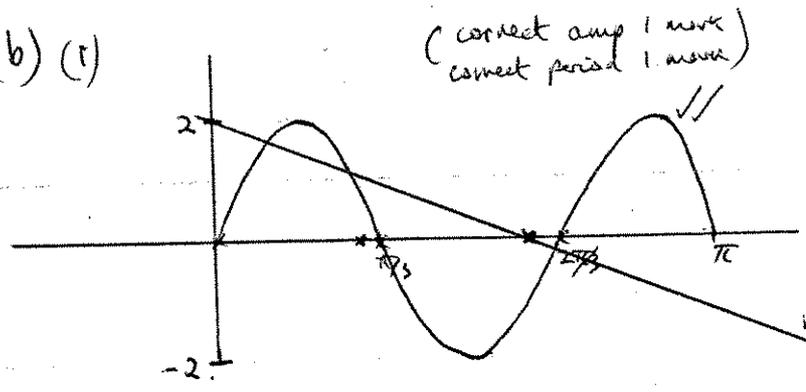
(ii)  $t = 0, 2, 6$

$t = 6$  since it has +ve velocity between  $t = 2$  and  $6$  for a greater time + to a greater degree than the negative  $V$  from  $t = 0$  to  $2$ .



(or area represents distance travelled  $0-2$  is neg  $2-6$  is +ve and greater)

(b) (i)



(correct amp 1 wave correct period 1 wave)

(ii) 5 solutions (3 in given domain)

(relative to trig graph)

(c)  $2\cos^2 x - 3\cos x - 2 = 0$   $0 \leq x \leq 2\pi$

$(2\cos x + 1)(\cos x - 2) = 0$

$\cos x = -\frac{1}{2}$  or  $\cos x = 2$

$\cos x = 2$  has no solution.

if  $\cos x = -\frac{1}{2}$ . (In Q2, 3 related  $< \pi/3$ )

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$$\text{Q(1)(a)} \quad y = \frac{1}{2}(e^x - e^{-x})$$

$$2y = e^x - \frac{1}{e^x}$$

$$2y = \frac{e^{2x} - 1}{e^x}$$

$$e^x 2y = e^{2x} - 1$$

$$e^{2x} - e^x 2y - 1 = 0 \quad \checkmark$$

$$e^x = \frac{2y \pm \sqrt{4y^2 + 4}}{2}$$

$$e^x = y \pm \sqrt{y^2 + 1} \quad \checkmark$$

Since  $e^x > 0$  and  $\sqrt{y^2 + 1} > y$

$$e^x = y + \sqrt{y^2 + 1} \quad \checkmark$$

$$x = \ln(y + \sqrt{y^2 + 1})$$

$$(b) \quad S = -kx^2 \ln x \quad k > 0.$$

$$\frac{dS}{dx} = -k \left( x^2 \cdot \frac{1}{x} + 2x \ln x \right)$$

$$= -kx(1 + 2 \ln x) \quad \checkmark$$

S max when  $\frac{dS}{dx} = 0$

$$\text{i.e. } -kx(1 + 2 \ln x) = 0$$

$$\therefore x = 0 \text{ or } 2 \ln x = -1$$

(since  $x \neq 0$  as  $S$  is undefined)

$$\ln x = -\frac{1}{2}$$

$$x = e^{-\frac{1}{2}}$$

$$\frac{d^2S}{dx^2} = -k \left( x + \frac{2}{x} + 1 + 2 \ln x \right) /$$

$$< 0 \quad \text{if } x = e^{-\frac{1}{2}}$$

$\therefore x = \frac{1}{\sqrt{e}}$  gives max value of  $S$ .  $\checkmark$

$$(c) i) 1 + 2 + 3 + \dots + 30.$$

$$\text{is AP } a=1 \quad l=30 \quad n=30$$

$$\text{Sum} = \frac{30(1+30)}{2}$$

$$= 465 \quad \checkmark$$

$$(ii) \frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots + \frac{n}{n}$$

$$= \frac{1+2+3+\dots+n}{n}$$

$$= \frac{\frac{n}{2}(n+1)}{n}$$

$$= \frac{n+1}{2} \quad \checkmark$$

$$(iii) \frac{1}{1} + \left[ \frac{1}{2} + \frac{2}{2} \right] + \left[ \frac{1}{3} + \frac{2}{3} + \frac{3}{3} \right] + \left[ \frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \frac{4}{4} \right] + \dots$$

$$N^{\text{th}} \text{ term } 1 + 2 + 3 + 4 + \dots$$

If there are 465 terms, then from part (i) above the final group is  $\left[ \frac{1}{30} + \frac{2}{30} + \dots + \frac{30}{30} \right]$  ✓

From (ii) each of these groups has a sum given by  $\frac{n+1}{2}$ .

$$\text{ie the series is } \frac{2}{2} + \frac{3}{2} + \frac{4}{2} + \dots + \frac{31}{2} \quad \checkmark$$

$$= 1 + 1\frac{1}{2} + 2 + \dots + 15\frac{1}{2}$$

$$\text{ie AP } a=1 \quad l=15\frac{1}{2} \quad n=30$$

$$\text{Sum} = \frac{30(1+15\frac{1}{2})}{2}$$

$$= 247\frac{1}{2} \quad \checkmark$$